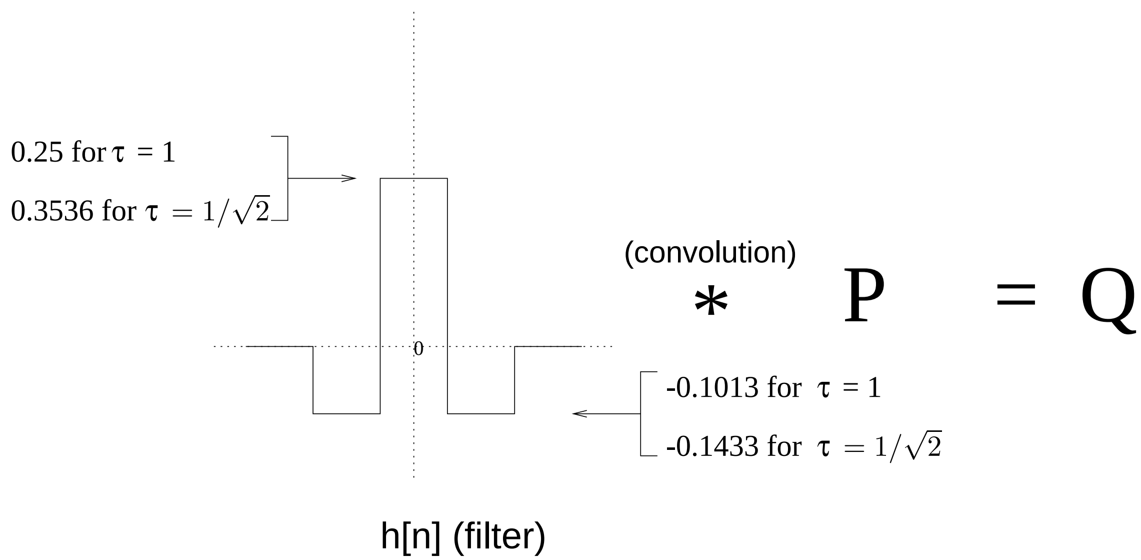
A simple numerical example of filtered back-projection

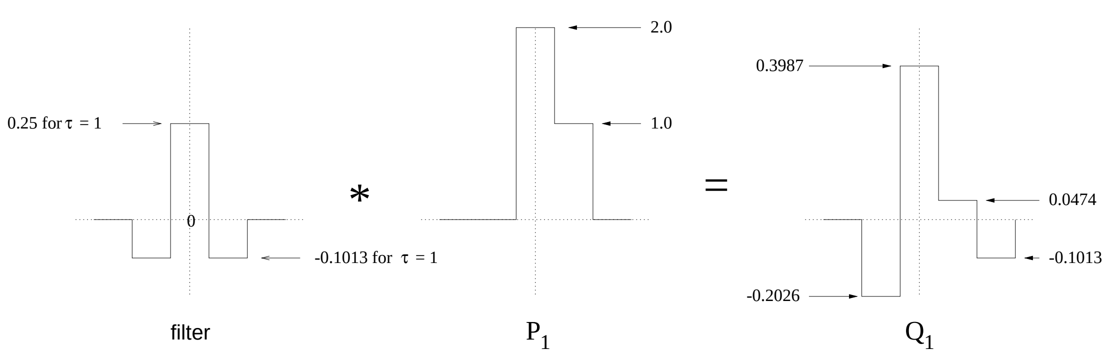
In the example that follows, for purposes of illustration and simplicity, we will assume

- the image space is a 5×5 (25 pixel) discrete array with the un-rotated reference coordinate system origin at the *centre* of the 5×5 pixel array.
- for projections at 0° and −90° the spatial resolution is one pixel (i.e $\tau = 1$), whereas for −45° and −135° the spatial resolution is at $1/\sqrt{2}$ pixel (i.e $\tau = 1/\sqrt{2}$).
- projections and filtered projections are plotted for the centre of each image pixel only
- that backprojection occurs only through the central portion of each image pixel

From our knowledge of the filtered backprojection algorithm backprojection follows filtering of the projection in order to "re-normalise" the wave-numbers i.e to re-emphasise the fine detail that was lost during the construction of the projections. This is illustrated diagrammatically below for a projection P.



Thus for P_1 of Figure 1 we obtain



Thus for the projections at $\theta = 0°$, $-45°$, $-90°$ and $-135°$ one obtains

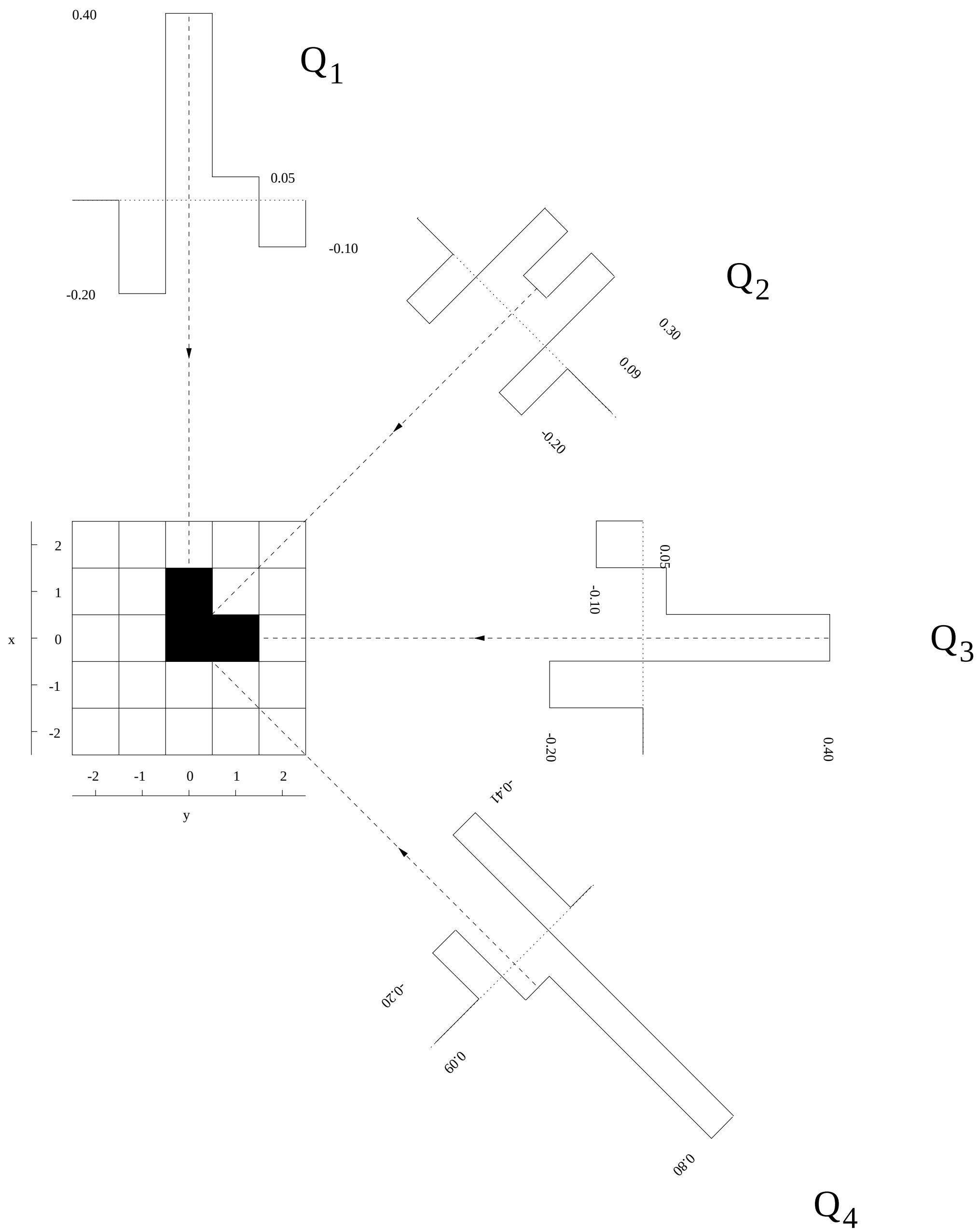


Figure 1: Filtered projections through 5×5 image array containing 3 squares having a linear attenuation coefficient of 1. All other image squares have a linear attenuation coefficient of zero. Projections are calculated assuming each ray passes through the *centre* of each square.

Backprojecting each filtered projection through the centre of each image pixel gives

	-0.20	0.40	0.05	-0.10
	-0.20	0.40	0.05	-0.10
	-0.20	0.40	0.05	-0.10
	-0.20	0.40	0.05	-0.10
	-0.20	0.40	0.05	-0.10

Q_1 back projected

		-0.20	0.30	0.09
	-0.20	0.30	0.09	0.30
-0.20	0.30	0.09	0.30	-0.20
0.30	0.09	0.30	-0.20	
0.09	0.30	-0.20		

Q_2 back projected

-0.10	-0.10	-0.10	-0.10	-0.10
0.05	0.05	0.05	0.05	0.05
0.40	0.40	0.40	0.40	0.40
-0.20	-0.20	-0.20	-0.20	-0.20

Q_3 back projected

0.09	0.80	-0.41		
-0.20	0.09	0.80	-0.41	
	-0.20	0.09	0.80	-0.41
		-0.20	0.09	0.80
			-0.20	0.09

Q_4 back projected

Thus adding all these backprojections together (first array), dividing by the number of projections (second array) and rounding in order to crudely eliminate unwanted values because of the limited number of projections, the original distribution of linear attenuation coefficients is reconstituted.

-0.01	0.50	-0.31	0.25	-0.11
-0.15	-0.26	1.55	-0.22	0.25
0.20	0.30	0.98	1.55	-0.31
0.10	-0.31	0.30	-0.26	0.50
0.09	0.10	0.20	-0.15	-0.01

$Q_1 + Q_2 + Q_3 + Q_4$

-0.01	0.39	-0.24	0.20	-0.09
-0.12	-0.20	1.22	-0.17	0.20
0.16	0.24	0.77	1.22	-0.24
0.08	-0.24	0.24	-0.20	0.39
0.07	0.08	0.16	-0.12	-0.01

$$\frac{\pi(Q_1 + Q_2 + Q_3 + Q_4)}{4}$$

see equation (40) IRFP notes

0	0	0	0	0
0	0	1	0	0
0	0	1	1	0
0	0	0	0	0
0	0	0	0	0

round

As an exercise the student may wish to repeat these steps by backprojecting unfiltered projections.