Answer to E4

Numerical solutions and properties of the Hodgkin–Huxley Equations

Mathew — SE404

Assuming that the steady–state intracellular concentration of K^+ (C_K^i) is 400 mmol L^{-1} what is the fractional change in C_K^i following a single action potential for a given stimulating current ? Assume that the model axon is 1 mm in diameter and 3.18 cm in length.

Axons have a high intracellular concentration of K^+ and the extracellular fluid has a low K^+ concentration. During an action potential there is an increase in K^+ permeability so there is a greater tendency for K^+ to flow out of the axon and into the extracellular space than there is at rest. The fractional change in K^+ will be

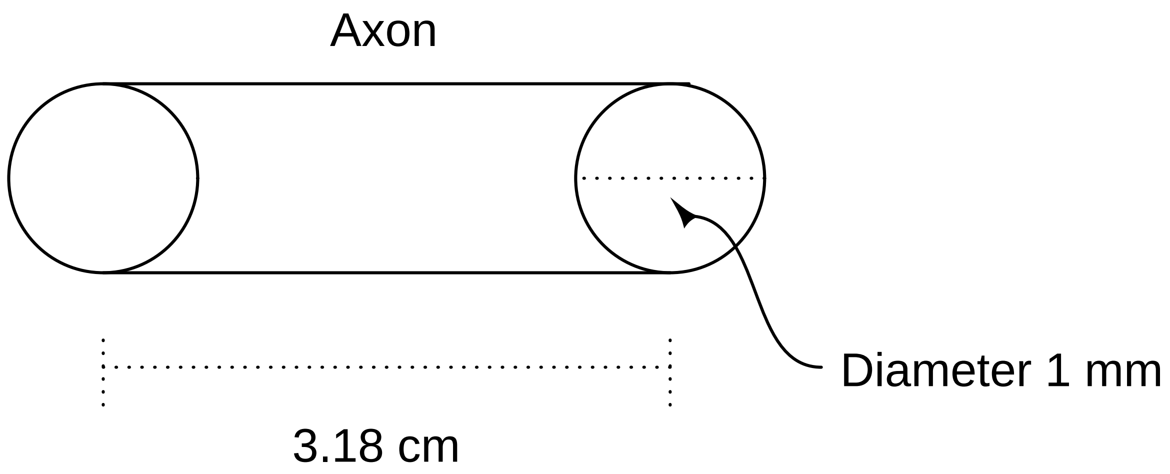
$$1 - \frac{K_i - K_{lost}}{K_i}$$

For a stimulating current of 6.96 μA cm^{-2} we know the total potassium charge flow (q_K from E3) is 1.4557×10^3 n C. So x the number of mol of K^+ ions that leak out during a single action potential is $x = q_K/F$ where q_K is the potassium charge flow and F (9.6487×10^4 C mol^{-1}) is the total charge per mol of ions of valence 1. So the number of potassium ions (in mol) that flow out of the axon during one action potential is

$$\begin{aligned} x &= \frac{1.4557 \times 10^3 \text{ n C}}{9.6487 \times 10^4 \text{ C mol}^{-1}} \\ &= 1.5087 \times 10^{-11} \text{ mol} \end{aligned}$$

How many K^+ ions were there in the axon to start with ? The volume of the axon is given by $V = \pi r^2 \ell$, so with r being 0.5 mm and ℓ being 3.18 cm our axonal volume is $2.49757 \times 10^{-8}\,\text{m}^3$. One litre is $1/1000\,\text{m}^3$ so the volume of the axon is $2.49757 \times 10^{-5}\,\text{L}$.

The number of mol of K^+ inside the axon is given by $n = cV$ where n is the number of mol, c is the concentration, and V is the volume of material. We know the concentration (400 milli mol L^{-1}) and we have just calculated the volume so the number of K^+ in mol is given by



$$
\begin{aligned}
n_{K^+} &= c\,V \\
&= \frac{400\,\text{milli mol}}{\text{L}} \times 2.49757 \times 10^{-5}\text{L} \\
&= 9.99026 \times 10^{-6}\,\text{mol}
\end{aligned}
$$

So the number of K^+ ions in the axon before the action potential is 9.99026×10^{-6} mol. The number of K^+ ions that flow out of the axon during the single action potential is 1.5087×10^{-11} mol. So the fractional change in C^i_K is

$$
\begin{aligned}
\Delta C^i_K &= 1 - \frac{K_i - K_{lost}}{K_i} \\
&= 1 - \frac{9.99026 \times 10^{-6}\,\text{mol} - 1.5087 \times 10^{-11}\,\text{mol}}{9.99026 \times 10^{-6}\,\text{mol}} \\
&= 1 - 0.9999984 \\
&= 1.51017 \times 10^{-6} \text{ or } 0.000151\%
\end{aligned}
$$

so overall very few potassium ions leak out of an axon during an action potential.