Tutorial Problem Sheet – Cable Equation

Tutorial time and place: 6/9/2004, 14:30–15:30, EN302

Answers to these problems will be collected **immediately** prior to the tutorial class. Problem sheets will under **no** circumstances be accepted following the tutorial.

1. Experiments at the turn of the 20th century indicated that for normal living cells the membrane electrical impedance was high for the flow of low frequency alternating current and low for high frequencies. These results suggested that the electrical characteristics of the cell membrane, for sinusoidal current, could be represented by a parallel conductance and capacitance. By the early 1920s experiments had established that the specific capacitance of a wide variety of cells was $\approx 1\ \mu\mathrm{F\,cm^{-2}}$.

   The early measurements of membrane capacitance allowed physiologists to infer the dimensions of the cell membrane by providing the first physico-chemical measurements of the thickness of the cell membrane.

   By assuming that the cell membrane is, to first approximation, essentially composed of two concentric and parallel sheets separated by a dielectric estimate the membrane thickness of a typical cell membrane based on the measurement given above. Assume that the dielectric constant $(\epsilon/\epsilon_0) \approx 3$ (characteristic of oils).

2. By defining the dimensionless quantities $X = x/\lambda_c$ and $T = t/\tau_m$, using the chain rule of differentiation and ignoring any external current input rewrite the cable equation (equation 17 – An Introduction to Core-conductor Theory, Class Notes) in a form that is free of any parameters.

3. Rewrite the cable equation (equation 17 – An Introduction to Core-conductor Theory, Class Notes) in terms of the following parameters;

   - R_m = membrane resistivity ($\Omega\,\mathrm{m}^2$) – the resistance of a unit area of cable membrane.
   - R_c = cytoplasmic resistivity ($\Omega\,\mathrm{m}$) i.e $r_i = R_c/A_i$ where A_i is the cross-sectional area of the cylindrical cable.

- C_m = membrane capacitance per unit area ($\mathrm{F\,m^{-2}}$).
- a = internal radius of the cable.

assuming negligible extracellular resistance and applied current only to the inner conductor.

Based on these results define τ_m and λ_c in terms of R_m, R_c, C_m and a.

4. A squid axon is placed in a large volume of seawater such that $r_o \ll r_i$. The following data are given: resistivity of squid axoplasm – $R_c = 30\,\Omega\,\mathrm{cm}$; diameter of squid axon – 500 μm; cable space constant – 6 mm; membrane thickness – 50 Å; capacitance per unit area – 1 $\mu\mathrm{F\,cm^{-2}}$.

   (a) Find the conductance of the axon per unit length – g_m.

   (b) Find the conductance of the axon per unit area – G_m.

   (c) Find λ_c for unmyelinated axons whose membranes have specific properties that are identical to those of the squid axon but whose radii are 0.25 mm, 25 μm, 2.5 μm and 0.25 μm.

   (d) Calculate τ_m for the squid axon and those considered above.

5. Consider a cable that extends from $X = 0$ to $X = \infty$ (where $X = x/\lambda_c$) in which a steady current I_{app} is applied to the inner conductor at $X = 0$ and V_m has attained its time independent state. Answer the following

   (a) Show that the boundary condition at $X = 0$ is

   $$\left.\frac{dV}{dX}\right|_{X=0} = -r_i \lambda_c I_{app}$$

   (b) Using the above boundary condition solve the time independent cable equation obtained from the result of Problem 2.

   (c) The input resistance R_{in} of a cable is defined to be the ratio $V(0)/I_{app}$. From the result of (b) show that

   $$R_{in} = \lambda_c r_i$$

   (d) Based on the results of (c) express R_{in} in terms of R_c, R_m and a (see problem 3). How does R_{in} vary as a function of a ?